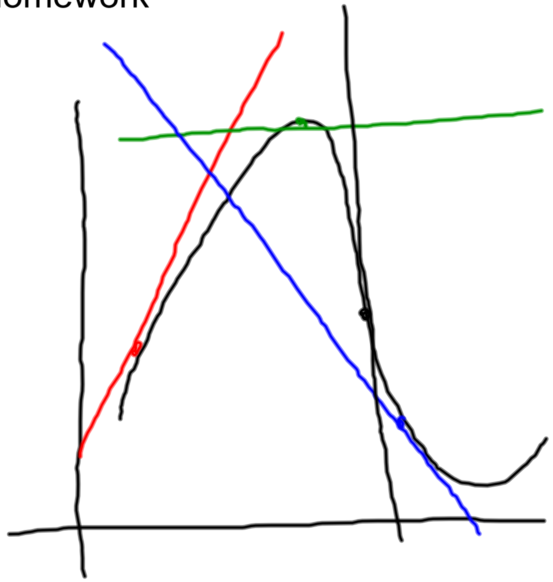


$f'(1)$ = the SLOPE of the tangent line at 1

$(0, 0.8), (1, 3)$

$\frac{3-0.8}{1-0} = 2.2$

$f'(3) = 0$

$(6,1)\ (5,3)$

$f'(5) = -2$

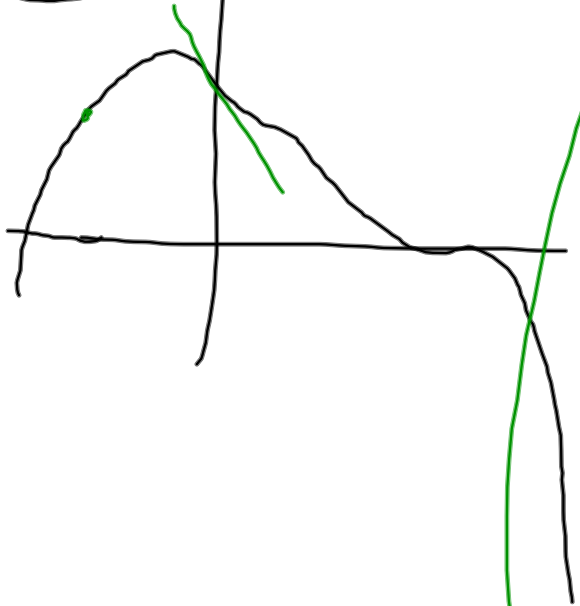
$(6,1), (0, 6.7)$

$f'(6) = \frac{6.7-1}{0-6} = \frac{5.7}{-6} = .95$

1-4, 9-12

9
1
4
10
12
3
2

2)



$f'(-3)$ 1ish

0

$f'(0)$ $-1 \Rightarrow -1.3$

$f'(2)$ $-\frac{1}{10}$

$f'(4)$ -4

3) a) have eq^n for tan @ $(a, f(a))$

$f'(a) =$ slope of the tan line

b) tan @ $(2,5)$ has eq^n $3x-1$

find $f'(2) = 3$

find $f'(3) =$ CAN'T

c) in part b, what is the instantaneous rate of chg of $f(x)$ @ $x=2$?

$= 3$

4) tan to $y=f(x)$ @ $(-1,3)$ passes thru $(0,4)$

$$f'(-1) = m = \frac{4-3}{0-(-1)} = \frac{1}{1} = 1$$

9) $f(x) = 3x^2$; $a=3$ use def 3.2.3

$$f'(x) = \lim_{w \to x} \frac{f(w)-f(x)}{w-x}$$

$$= \lim_{w \to x} \frac{3w^2 - 3x^2}{w-x}$$

$3(w^2-x^2)$

$$= \lim_{w \to x} \frac{3(w-x)(w+x)}{(w-x)}$$

$$= \lim_{w \to x} 3(w+x) = 3(2x) = 6x$$

b) eqn of tan @ $x=3$

$$f'(3) = 6(3) = 18$$

$$Pt = (3, f(3))$$

$$= (3, 3(3^2))$$

$$= (3, 27)$$

Eqn:

$$y - 27 = 18(x-3)$$

3.2.2

$$f'(x_0) = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

slope of tan line @ $x = x_0$

3.2.3

$$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

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20

15 - 18

23

15) use formula 13

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$y = \frac{1}{x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{\overset{x}{(x)} - \overset{-x \; -\Delta x}{(x+\Delta x)}}{x(x+\Delta x)}\left(\frac{1}{\Delta x}\right)}{\Delta x \left(\frac{1}{\Delta x}\right)} = \lim_{\Delta x \to 0} \frac{-(\Delta x)}{x(x+\Delta x)(\Delta x)}$$

$$\frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{bc}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = \lim_{\Delta x \to 0} \frac{-1}{x^2} = \frac{-1}{x^2}$$

16 $y = \dfrac{1}{x+1}$ $\qquad \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\dfrac{1}{(x+\Delta x)+1} - \dfrac{1}{(x)+1}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{(x+1) - (x+\Delta x+1)}{(x+1)(x+\Delta x+1)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{-\Delta x}{(x+1)(x+\Delta x+1)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{-1}{(x+1)(x+\Delta x+1)}$$

$$= \frac{-1}{(x+1)^2}$$

LIMIT $\dfrac{dy}{dx}$ = $\dfrac{\text{instantaneous difference in } y}{\text{instantaneous difference in } x}$

infinitesmally small diff in y

infinitesimally small diff in x

avg rate of chg $= \dfrac{\Delta y}{\Delta x}$

$= \dfrac{\text{difference in } y}{\text{difference in } x}$ mod

Newton Leibniz

derivation fluxion derivative

$\dot{f}(x)$ $\dfrac{dy}{dx}$ $f'(x)$

$f'(x) = $ "second derivative" $\dfrac{d^2y}{dx^2}$

$= $ derivative of the first derivative $f''(x)$ $f^{(2)}(x)$

"Limit Definitions of a Derivative"

$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$\lim_{w \to x} \frac{f(w)-f(x)}{w-x}$$

$$= \frac{dy}{dx} = \frac{d}{dx}\left(f(x)\right) = f'(x)$$

slope of tangent line

inst. rate of chg

1	2	3	4	5
Hw Unbearably difficult				Hw EASY

time spent each day (avg)

1		5
bored stiffly		learning a lot

Spoiler Alert (3.3)

$$y = x^n \text{ then } \frac{dy}{dx} = nx^{(n-1)}$$

$$\frac{d}{dx}(3x^4)$$
$$= 3\frac{d}{dx}(x^4)$$
$$= 3(4x^3) = 12x^3$$

$$\frac{d}{dx}(x^{1/2})$$
$$= \frac{1}{2}x^{\frac{1}{2}-1}$$
$$= \frac{1}{2}x^{-\frac{3}{2}}$$
$$= \frac{-1}{2x^{3/2}}$$

$$y = x^{-1} \text{ then } \frac{dy}{dx} = (-1)x^{-1-1} = (-1)x^{-2}$$

$$y = x^3 \text{ then } \frac{dy}{dx} = 3x^2$$

$$y = x^{1/2} \text{ then } \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx}(x^2 - x)$$
$$= \frac{d}{dx}(x^2) - \frac{d}{dx}(x^1)$$
$$2x^1 - 1x^0$$
$$= 2x - 1$$

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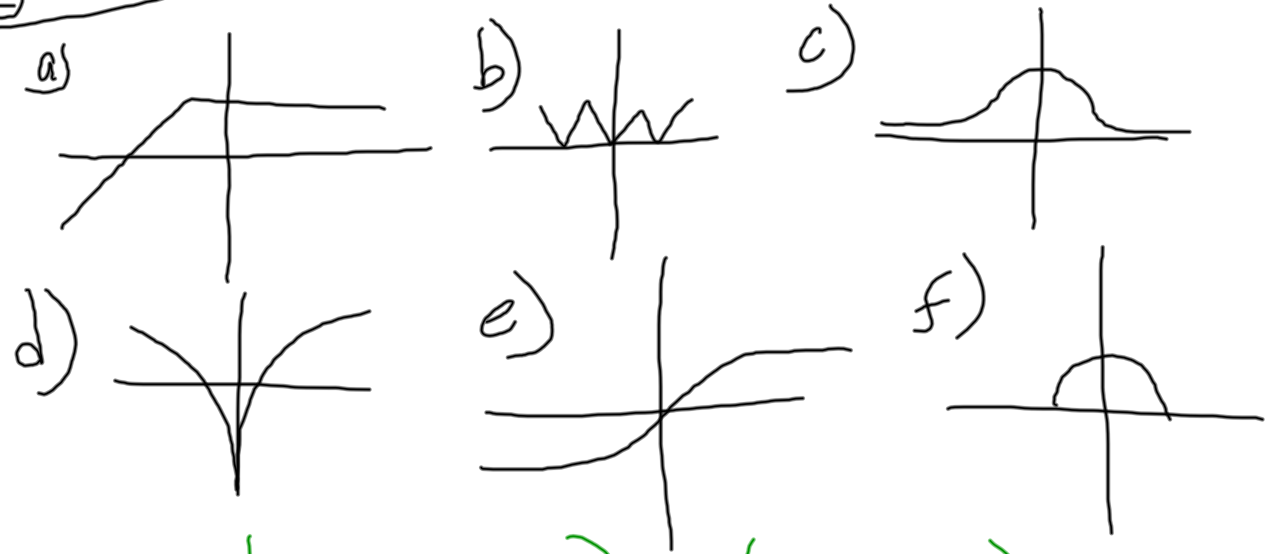
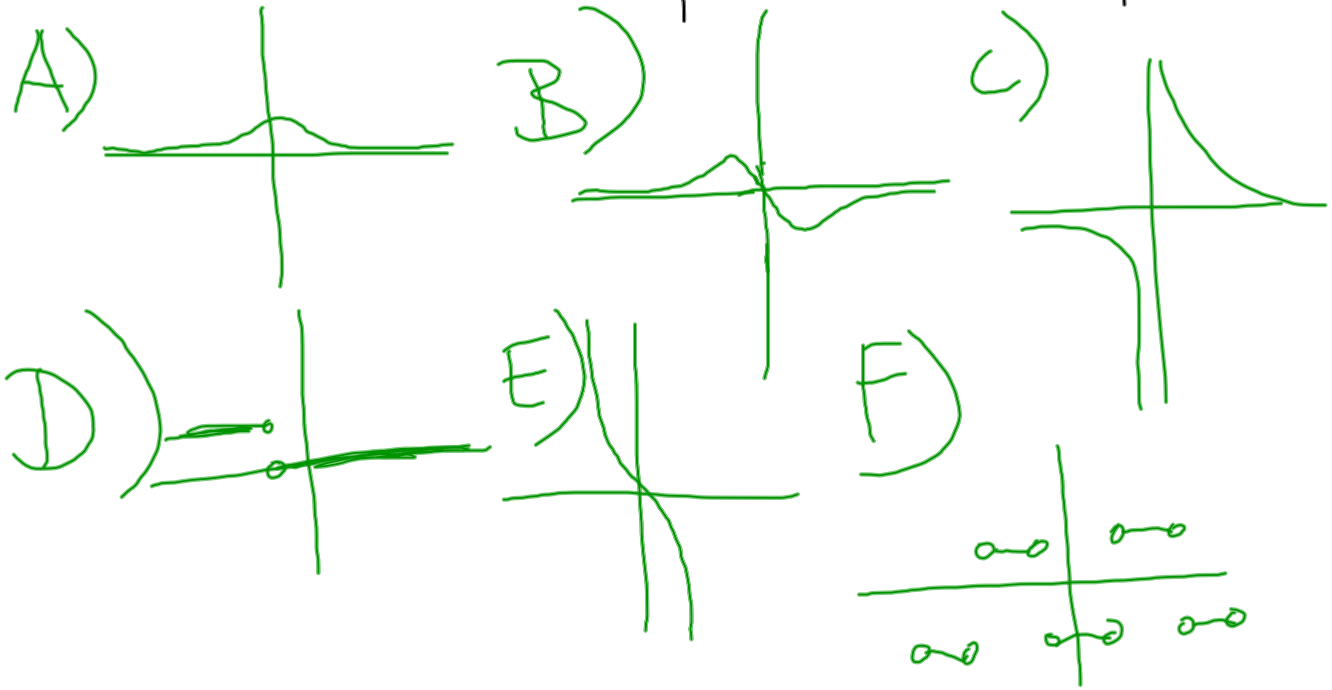
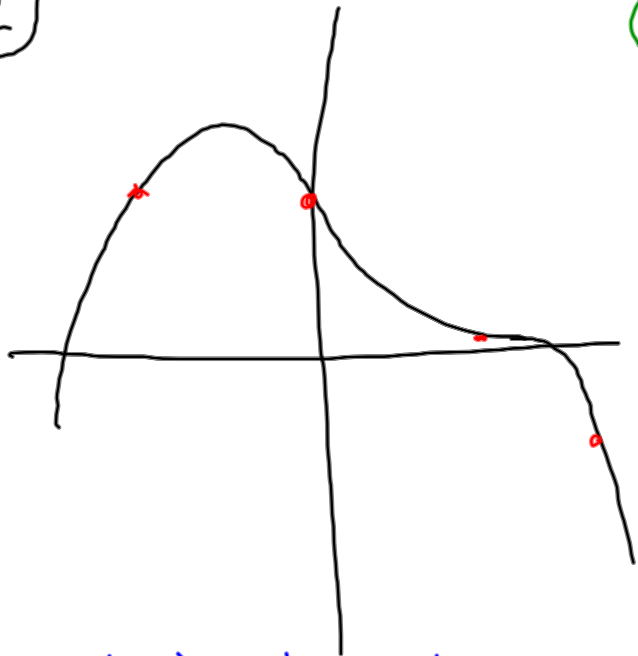




1-4, 9-12

2
12
9
10, 11

2)



⑤ $f'(-3)$ 1

④ 0 0

② $f'(0)$ −1

③ $f'(2)$ $-\frac{3}{1}$? , $-\frac{1}{8}$!

① $f'(4)$ −3,

$f'(-3)$ = derivative at $x=-3$

= slope of the tangent line to f at $x=-3$

= instantaneous r.o.c. of f (wrt) x at $x=-3$

9) $f(x)=3x^2$; $a=3$

(a) use 3.2.3

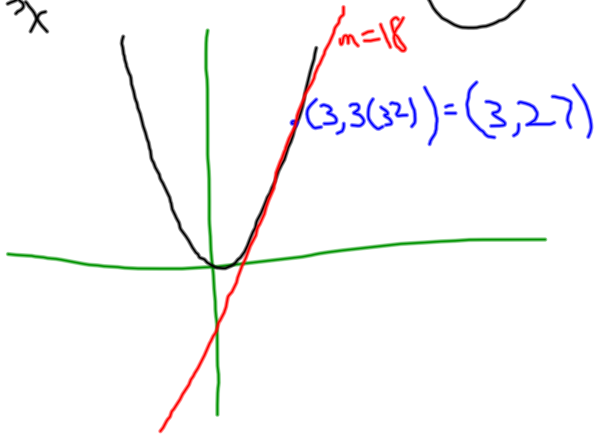
$$f'(x)=\lim_{w\to x}\frac{f(w)-f(x)}{w-x}$$

$$=\lim_{w\to x}\frac{3w^2-3x^2}{w-x}$$

$3(w^2-x^2)$

$$=\lim_{w\to x}\frac{3(w-x)(w+x)}{(w-x)}$$

$$=\lim_{w\to x}3(w+x)=3(2x)=6x$$



(b) write eqⁿ of a tan line @ $x=3$

P_T: $(3, f(3))$

$$=(3,3(3^2))=(\overset{x_0}{3},\overset{y_0}{27})$$

slope $=f'(3)=6(3)=18$ m

Eqⁿ:

$$y-y_0=m(x-x_0)$$

$$y-27=18(x-3)$$

10) $f(x) = x^4$; $a = -2$

(a) $$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$= \lim_{w \to x} \frac{w^4 - x^4}{w - x}$$

$(w^2 - x^2)(w^2 + x^2)$

$(w - x)(w + x)$

$$= \lim_{w \to x} \frac{(w-x)(w^3 + w^2x + wx^2 + x^3)}{(w-x)}$$

$$= \lim_{w \to x} (w^3 + w^2x + wx^2 + x^3) = x^3 + x^3 + x^3 + x^3$$

$$= 4x^3$$

b) $f'(-2) = 4(-2)^3$

$= 4(-8)$

$= -32$

Pt: $(-2, f(-2))$

$= (-2, 16)$

$(-2)^4$

$m = -32$

Eq:

$y - 16 = -32(x - (-2))$

11) $f(x) = x^3$; $a = 0$

ⓐ $$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$= \lim_{w \to x} \frac{w^3 - x^3}{w - x}$$

$$= \lim_{w \to x} \frac{(w-x)(w^2 + xw + x^2)}{(w-x)}$$

$$= \lim_{w \to x} (w^2 + xw + x^2) = x^2 + x^2 + x^2$$

$$= 3x^2$$

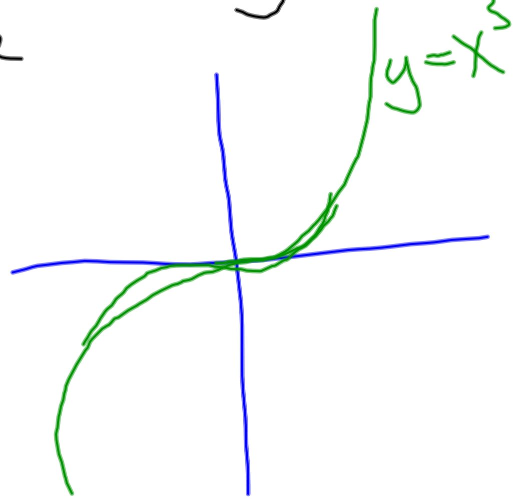
ⓑ $f'(0) = 3(0)^2 = 0$

Pt: $(0, f(0)) = (0, 0)$

$m = f'(0) = 0$

Eq: $y - 0 = 0(x - 0)$

$y = 0$



12) $f(x) = 2x^3 + 1$; $a = -1$

a) $$f'(x) = \lim_{w \to x} \frac{f(w) - f(x)}{w - x}$$

$$= \lim_{w \to x} \frac{(2w^3 + 1) - (2x^3 + 1)}{w - x}$$

$$= \lim_{w \to x} \frac{2w^3 - 2x^3}{w - x}$$

$$= \lim_{w \to x} \frac{2(w-x)(w^2 + xw + x^2)}{w - x}$$

$$= \lim_{w \to x} 2(w^2 + xw + x^2) = 2(x^2 + x^2 + x^2) = 6x^2$$

b) $$f'(-1) = 6(-1)^2$$
$$= 6$$

$$Pt = (-1, f(-1))$$
$$= (-1, 2(-1)^3 + 1)$$
$$= (-1, -1)$$

$m = f'(-1) = 6$

Eq: $y - (-1) = 6(x - (-1))$

3.2) 13) $f(x)=\sqrt{x+1}$; $a=8$

a) $f'(x)=\lim\limits_{w\to x}\dfrac{\sqrt{w+1}-\sqrt{x+1}}{w-x}$

$=\lim\limits_{w\to x}\dfrac{(\sqrt{w+1}-\sqrt{x+1})(\sqrt{w+1}+\sqrt{x+1})}{(w-x)(\sqrt{w+1}+\sqrt{x+1})}$

$=\lim\limits_{w\to x}\dfrac{(w+1)-(x+1)}{(w-x)(\sqrt{w+1}+\sqrt{x+1})}$

$=\lim\limits_{w\to x}\dfrac{w-x}{(w-x)(\sqrt{w+1}+\sqrt{x+1})}=\dfrac{1}{2\sqrt{x+1}}$

b) $f'(8)=\dfrac{1}{2\sqrt{8+1}}=\dfrac{1}{6}$

Pt: $(8,f(8))=(8,\sqrt{9})=(8,3)$

Eqn: $y-3=\frac{1}{6}(x-8)$

14) $f(x)=\sqrt{2x+1}$; $a=4$

a) $f'(x)=\lim\limits_{w\to x}\dfrac{\sqrt{2w+1}-\sqrt{2x+1}}{w-x}$

$=\lim\limits_{w\to x}\dfrac{(2w+1)-(2x+1)}{(w-x)(\sqrt{2w+1}+\sqrt{2x+1})}$

$=\lim\limits_{w\to x}\dfrac{2(w-x)}{(w-x)(\sqrt{2w+1}+\sqrt{2x+1})}$

$=\dfrac{2}{2\sqrt{2x+1}}=\dfrac{1}{\sqrt{2x+1}}$

b) $f'(4)=\dfrac{1}{3}$

Pt: $(4,f(4))=(4,3)$

Eqn: $y-3=\frac{1}{3}(x-4)$

15) $y = \frac{1}{x}$ use (13)

$$\boxed{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

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18–19
23
17
15

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)} - \frac{1}{x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{x-(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{-\Delta x}{x(x+\Delta x)}}{\underset{1}{\Delta x}} = \lim_{\Delta x \to 0} \frac{-\cancel{(\Delta x)}}{x(x+\Delta x)} \cdot \frac{1}{\cancel{(\Delta x)}}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)} = -\frac{1}{x^2}$$

17) $y = ax^2 + b$ $\qquad bx^0 \;\longrightarrow\; 2ax^1 + 0$

$\qquad\qquad f(x+\Delta x) \qquad\qquad - \; f(x)$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\left[a(x+\Delta x)^2 + b\right] - \left[ax^2 + b\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{a\left(x^2 + 2x\,\Delta x + (\Delta x)^2\right) + b - \left(ax^2 + b\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2ax\,\Delta x + a(\Delta x)^2}{\Delta x} = \lim_{x \to 0} \frac{\Delta x\left(2ax + a\Delta x\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(2ax + a\Delta x\right) = 2ax$$

Local Linear Approximation

= approx. function w/ tangent line

$$y = \frac{1}{\sqrt{x}} \qquad \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{1}{\sqrt{x+\Delta x}} - \frac{1}{\sqrt{x}}}{\Delta x} = \lim_{\Delta x \to 0} \frac{\frac{\sqrt{x} - \sqrt{x+\Delta x}}{\sqrt{x}\sqrt{x+\Delta x}}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{\sqrt{x} - \sqrt{x+\Delta x}}{\sqrt{x}\sqrt{x+\Delta x}}}{\Delta x} \left(\frac{\sqrt{x}+\sqrt{x+\Delta x}}{\sqrt{x}+\sqrt{x+\Delta x}}\right) = \lim_{\Delta x \to 0} \frac{\frac{\overbrace{(x) - (x+\Delta x)}^{-\Delta x}}{(\sqrt{x})\sqrt{x+\Delta x}\left(\sqrt{x}+\sqrt{x+\Delta x}\right)}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\Delta x}{\sqrt{x}\sqrt{x+\Delta x}\left(\sqrt{x}+\sqrt{x+\Delta x}\right)\Delta x}$$

$$\frac{d}{dx} x^{-1/2} = -\frac{1}{2} x^{-3/2}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{\sqrt{x}\sqrt{x+\Delta x}\underbrace{\left(\sqrt{x}+\sqrt{x+\Delta x}\right)}_{(2\sqrt{x})}} = \frac{-1}{\sqrt{x}\sqrt{x}\sqrt{x}} = \frac{-1}{2x^{3/2}}$$

$$= -\frac{x^{-3/2}}{2}$$

$$= \frac{-1}{2x\sqrt{x}}$$